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Vector potential quantization and the photon intrinsic electromagnetic properties: Towards nondestructive photon detection

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We employ here the enhancement of the vector potential amplitude quantization at a single photon state. The analysis of the general solution of the vector potential, obtained by resolving Maxwell's equations, implies that the amplitude is proportional to the angular frequency. The photon vector potential function $\boldsymbol{\alpha}_{k\lambda}(\mathbf{r}, t)$ can be written in the plane wave representation satisfying the classical wave propagation equation, Schrödinger's equation for the energy with the relativistic massless field Hamiltonian and a linear time-dependent equation for the vector potential amplitude operator. Thus, the vector potential $\boldsymbol{\alpha}_{k\lambda}(\mathbf{r}, t)$ with the quantized amplitude may play the role of a real wave function for the photon in a nonlocal representation that can be suitably normalized. We then deduce that the amplitudes of the electric and magnetic fields, respectively, of a single free photon are proportional to the square of the angular frequency. This might open perspectives for the development of nondestructive photon detection methods based on the influence of the electric and/or magnetic fields of photons on the energy levels of atoms and molecules.

Keywords: Electromagnetic field; vector potential quantization; photons; photon wave function; photon wave-particle equation; photon electric field.

1. Introduction

In recent years, the experimental evidence of the entangled states produced with single photons sources has demonstrated the permanent violation of Bell's inequality entailing the necessity of developing a nonlocal representation for the photon, through a real wave function.[1–8] At the same time, the existence of hidden variables within a local representation is rejected while new variables in a nonlocal representation are not explicitly excluded.[3,6,9] Hence, we investigate in this paper, the possibility of expressing the single free photon intrinsic electromagnetic properties within a nonlocal representation[10–13] through the vector potential with quantized amplitude. In fact, a detailed unit analysis of the general solution of Maxwell's equations for the vector potential shows[14–16] that the amplitude is proportional to the angular frequency. Thus, for a k-mode photon with angular frequency ω_k, we may write the vector potential amplitude as $\alpha_{0k} = \xi\omega_k$, where ξ is a constant.

In a first approximation, the value of the constant ξ has been evaluated[14,16–18] by normalizing the mean energy of a plane electromagnetic wave over a wavelength to a single photon energy getting

$$|\xi| \approx \left| \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\hbar}{8\alpha_{fs}\varepsilon_0 c^3}} \right| = \left| \frac{\hbar}{4\pi e c} \right| = 1.747 \times 10^{-25}\,\mathrm{V\,m^{-1}s^2} \tag{1}$$

with $\alpha_{fs} = 1/137$ the fine structure constant, ε_0 the vacuum electric permittivity, c the speed of light in vacuum and e the electron charge.

The fundamental physical properties characterizing the wave-particle nature of a single k-mode photon[14–18]: {energy E_k and momentum $\mathbf{p}_k$} (particle), {vector potential amplitude α_{0k}, wave vector $\mathbf{k}$ and dispersion relation} (wave), are all related directly to the angular frequency ω_k

$$\frac{E_k}{\hbar} = \frac{|\mathbf{p}_k|}{\hbar/c} = \frac{\alpha_{0_k}}{\xi} = |\mathbf{k}|c = \omega_k. \tag{2}$$

The last relation expresses the wave-particle properties of a single free photon state introducing the electromagnetic nature through the quantized vector potential amplitude.

2. Photon Vector Potential Within Nonlocal Representation and Intrinsic Electromagnetic Fields

The relation (2) entails that the quantized vector potential amplitude is an intrinsic property of a single photon. Hence, the vector potential for a k-mode and λ-polarization photon can now be written in the plane wave representation

$$\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t) = \omega_k(\xi\hat{\varepsilon}_\lambda e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_k t+\theta)} + \xi * \hat{\varepsilon}^*_\lambda e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_k t+\theta)}), \tag{3}$$

where λ takes two values corresponding to the left (L) and right (R) circular polarizations expressed by the unit vector ε_λ.

The general expression for the vector potential of an electromagnetic wave considered as a superposition of different modes writes

$$\mathbf{A}(\mathbf{r},t) = \sum_{k,\lambda} \omega_k[\xi\hat{\varepsilon}_\lambda e^{i(\omega_k t-\mathbf{k}\cdot\mathbf{r}+\varphi)} + \xi^*\hat{\varepsilon}^*_\lambda e^{-i(\omega_k t-\mathbf{k}\cdot\mathbf{r}+\varphi)}]. \tag{4}$$

It can be easily shown[14,16,18,19] that the vector potential function (3) satisfies the classical wave propagation equation in vacuum

$$\boldsymbol{\nabla}^2\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{\alpha}_{\kappa,\lambda}(\mathbf{r},t) = 0 \tag{5}$$

as well as Schrödinger's equation with the well-known relativistic massless particle Hamiltonian $\tilde{H} = -i\hbar c\boldsymbol{\nabla}$

$$i\hbar \frac{\partial}{\partial t}\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t) = \tilde{H}\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t) \tag{6}$$

having the eigenvalue of a single photon energy.

The vector potential function (3) also satisfies a linear time-dependent equation for the vector potential amplitude operator[14–16]

$$\tilde{\alpha}_0 = -i\xi c\boldsymbol{\nabla} \tag{7}$$

which writes

$$i\xi \frac{\partial}{\partial t}\boldsymbol{\alpha}_{k;\lambda}(\mathbf{r},t) = \tilde{\alpha}_0\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t). \tag{8}$$

The symmetrical equations for the energy and the vector potential, (6) and (8), respectively, can be combined to obtain the *vector potential* — *energy* (*wave-particle*) equation for the photon

$$i\begin{pmatrix}\xi\\ \hbar\end{pmatrix}\frac{\partial}{\partial t}\boldsymbol{\alpha}_{k;\lambda}(\mathbf{r},t) = \begin{pmatrix}\tilde{\alpha}_0\\ \tilde{H}\end{pmatrix}\boldsymbol{\alpha}_{k,\lambda}(\mathbf{r},t) \tag{9}$$

having the eigenvalues $\xi\omega_k$ and $\hbar\omega_k$, respectively.

The photon as an integral entity is not a point but extends naturally over a wavelength according to the experimental evidence[3,20,21] in agreement with the density of states theory.[3,18,22–24] The wave function of the quantized vector potential (3) ensures that the photon as a whole propagates along the propagation axis with circular (L or R) polarization rotating at the angular frequency ω_k.

For a cavity free k-mode photon, we can deduce the amplitude of the intrinsic electric field $|\boldsymbol{\varepsilon}_{k\lambda}|$ which appears to be proportional to the square of the angular frequency

$$|\boldsymbol{\varepsilon}_{k\lambda}| = \left|-\frac{\partial}{\partial t}\boldsymbol{\alpha}_{k\lambda}(\mathbf{r},t)\right| \propto \xi\omega_k^2. \tag{10}$$

According to the plane wave representation, the corresponding magnetic field $|\boldsymbol{\beta}_{k\lambda}|$ of the free photon is equally proportional to the square of the angular frequency

$$|\boldsymbol{\beta}_{k\lambda}| \propto \sqrt{\varepsilon_0\mu_0}\xi\omega_k^2, \tag{11}$$

where μ_0 is the vacuum magnetic permeability.

It is of fundamental interest to investigate experimentally the last results. High resolution laser spectroscopy might be employed for the measurement of the Stark shifts induced by the electric field of an energetic photon (e.g. in the UV range) on the atomic or molecular energy levels. Such an experimental investigation could also confirm the dependence of the single photon electric field on the square of the angular frequency. In fact, the energy shift due to the linear Stark effect δE_{Stark} is directly proportional to the local electric field strength which for the photon depends on the angular frequency, $\delta E_{\text{Stark}} \propto |\boldsymbol{\varepsilon}_{k\lambda}(\omega_k)| \propto \omega_k^p$. The ratio of the measured Stark shifts $\delta E_{\text{Stark}}^{(1)}$ and $\delta E_{\text{Stark}}^{(2)}$, induced independently by two photons with quite different energies $\hbar\omega_{k1}$ and $\hbar\omega_{k2}$, could permit to define directly the dependence of the photon electric field on the angular frequency according to the expression $\frac{\delta E_{\text{Stark}}^{(1)}}{\delta E_{\text{Stark}}^{(2)}} \propto (\frac{\omega_{k1}}{\omega_{k2}})^p$. That is, e.g. if $p = 2$, as deduced by the relation (10), then for $\omega_{k1} = 2\omega_{k2}$, the ratio of the measured Stark shifts should be roughly $\delta E_{\text{Stark}}^{(1)} \approx 4\delta E_{\text{Stark}}^{(2)}$. Such an experimental confirmation might also open perspectives for the development of nondestructive photon detection methods through the measurement of the influence of the photon electric or magnetic fields upon the atomic or molecular energy levels.

3. Conclusion

Within a nonlocal photon representation, as required by entangled states experiments, the vector potential function $\boldsymbol{\alpha}_{k\lambda}(\mathbf{r}, t)$ with the quantized amplitude $\alpha_{0k}(\omega_k) = \xi\omega_k$ can play the role of a real wave function for the photon that can be suitably normalized. A k-mode photon as a quantum of the electromagnetic field and as an integral physical entity subsists over a period (thus over a wavelength) through the oscillation of the quantized vector potential at the angular frequency ω_k and propagates along the propagation axis according to the wave function $\boldsymbol{\alpha}_{k\lambda}(\mathbf{r}, t)$. At any point within a wavelength the oscillation of the vector potential gives birth to intrinsic electric and magnetic fields, also oscillating at the same frequency and whose strength is proportional to the square of the angular frequency. This fundamental physical aspect needs to be further investigated experimentally. Following these results, nondestructive photon detection methods might be developed based on the detection of the influence of the photon intrinsic electric and/or magnetic fields upon the matter.

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